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Neutrosophic Cubic MCGDM Method Based on Similarity Measure

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Abstract. The notion of neutrosophic cubic set is originated from the hybridization of the concept of neutrosophic set and interval valued neutrosophic set. We define similarity measure for neutrosophic cubic sets and prove some of its basic properties. We present a new multi criteria group decision making method with linguistic variables in neutrosophic cubic set environment. Finally, we present a numerical example to demonstrate the usefulness and applicability of the proposed method.

Keywords: Cubic set, Neutrosophic cubic set, similarity measure, multi criteria group decision making.

1. Introduction

In practical life we frequently face decision making problems with uncertainty that cannot be dealt with the classical methods. Therefore sophisticated techniques are required for modification of classical methods to deal decision making problems with uncertainty. L. A. Zadeh [1] first proposed the concept of fuzzy set to deal non-statistical uncertainty called fuzziness. K. T. Atanassov [2, 3] introduced the concept of intuitionistic fuzzy set (IFS) to deal with uncertainty by introducing the non-membership function as an independent component. F. Smarandache [4, 5, 6, 7, 8] introduced the notion of neutrosophic set by introducing indeterminacy as independent component. The theory of neutrosophic sets is a powerful tool to deal with incomplete, indeterminate and inconsistent information involed in real world decision making problem. Wang et al. [9] defined single valued neutrosophic set (SVNS) which is an instance of neutrosophic set. SVNS can independently express a truth-membership degree, an indeterminacy-membership degree and non-membership (falsity-membership) degree. SVNS is capable of representing human thinking due to the imperfection of knowledge received from real world problems. SVNS is obviously suitable for representing incomplete, inconsistent and indeterminate information.

Neutrosophic sets and SVNSs have become hot research topics in different areas of research such as conflict resolution [10], clustering analysis [11, 12], decision making [13-41], educational problem [42, 43], image processing [44, 45, 46], medical diagnosis [47], optimization [48-53], social problem [54, 55].

By combining neutrosophic sets and SVNS with other sets, several neutrosophic hybrid sets have been proposed in the literature such as neutrosophic soft sets [56, 57, 58, 59, 60, 61], neutrosophic soft expert set [62, 63], single valued neutrosophic hesitant fuzzy sets [64, 65, 66, 67, 68], interval neutrosophic hesitant sets [69], interval neutrosophic linguistic sets [70], single valued neutrosophic linguistic sets [71], rough neutrosophic set [72, 73, 74, 75, 76, 77, 78, 79], interval rough neutrosophic set [80, 81, 82], bipolar neutrosophic set [83, 84], bipolar rough neutrosophic set [85] Tri-complex rough neutrosophic set[86], hyper complex rough neutrosophic set [87]. Neutrosophic refined set [88, 89, 90, 91, 92, 93], Bipolar neutrosophic refined sets [94], rough complex set neutrosophic cubic set [95].

Jun et al. [96] put forward the concept of cubic set in fuzzy environment and defined external and internal cubic set. Ali et al. [95] proposed neutrosophic cubic set and defined external and internal neutrosophic cubic sets and their basic properties.

Similarity measure is a vital topic in fuzzy set theory, Chen and Hsiao [97] presented comparisons of similarity measures of fuzzy sets. Pramanik and Mondal [98] studied weighted fuzzy similarity measure based on tangent function and presented its application to medical diagnosis. Hwang and Yang [99] constructed a new similarity measure between intuitionistic fuzzy sets based on lower, upper and middle fuzzy sets. Pramanik and Mondal [100] developed tangent similarity measures in intuitionistic fuzzy environment and applied to medical diagnosis. Ren and Wang [101] proposed similarity measures in interval-valued intuitionistic fuzzy environment and applied it to multi attribute decision making problems. Baccour et al. [102] presented survey of similarity measures for intuitionistic fuzzy sets. Baroumi and Smarandache [103] disicussed several similarity measures of neutrosophic sets. Mondal and Pramanik [104] extended the concept of intuitionistic tangent similarity measure to neutrosophic environment. Biswas et al. [105] studied cosine similarity measure with trapezoidal fuzzy neutrosophic number and its applied to multi attribute decision making problems. Pramanik and Mondal [106] proposed cosine similarity measure of rough neutrosophic set and applied it to medical diagnosis problems. Pramanik and Mondal [107]developed ccotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis J. Ye [108] proposed a similarity measures under interval neutrosophic domain using hamming distance and Euclidean distance. P. Majumdar and S. K. Samanta [109] introduced some measures of similarity and entropy of single valued neutrosophic sets. Ali aydogdu [110] proposed similarity and entropy measure of single valued neutrosophic sets. Ali aydogdu [111] also defined entropy and similarity measures of interval neutrosophic sets. Mukherjee and Sarkar [112] proposed similarity measures, weighted similarity measure and developed an algorithm in interval valued neutrosophic soft set setting for supervised pattern recognition problem. In neutrosophic cubic set environment, similarity measure is yet to appear.

In this paper we define similarity measures in neutrosophic cubic set environment and develop a multi criteria group decision making (MCGDM) method in neutrosophic cubic set setting. The decision makers' weights and criteria (attributes) weights are described by neutrosophic cubic numbers using linguistic variables. The ranking of alternatives is presented in descending order. Finally, illustrate numerical example MCGDM problem in neutrosophic cubic set environment is dolved to show the effectiveness of the proposed method.

Rest of the paper is presented as follows. Section 2 presents some basic definition of fuzzy sets, interval-valued fuzzy sets, neutrosophic sets, interval valued neutrosophic sets, cubic set, neutrosophic cubic sets and their basic operations. Section 3 is devoted to prove the basic properties of similarity measure for neutrosophic cubic sets. Section 4 presents a MCGDM method based on similarity measure in neutrosophic cubic set environment. Section 5 presents a numerical example for a MCGDM problem. Finally, section 6 presents conclusion and future scope of research.

2 Preliminaries

In this section, we recall some basic definitions which are relevant to develop the paper.

Definition 2.1 [1] Fuzzy set

Let U be a universal set. Then a fuzzy set Z over U is defined by Z = $\{(u, \mu_Z(u)): u \in U\}$

Where $\mu_Z : U \rightarrow [0, 1]$ is called membership function of Z and $\mu_Z(u)$ specifies the grade or degree to which any element u in Z, $\mu_Z(u) \in [0, 1]$. Larger values of $\mu_Z(u)$ indicate higher degrees of membership.

Definition 2.2 [113] Interval valued fuzzy set

Let U be a universal set, then an interval valued fuzzy set $\tilde{Z}$ over U is defined by $\tilde{Z} = \{[Z^-(u), Z^+(u)] / u: u \in U\}$, where $Z^-(u)$, $Z^+(u)$ represent respectively the lower and upper degrees of membership values for $u \in U$ and $0 \leq Z^-(u) + Z^+(u) \leq 1$.

Definition 2.3 [96] Cubic set

Let G be a non-empty set. A cubic set C (G) in G is defined by

C (G) = $\{g, \tilde{Z}(g), Z(g) / g \in G\}$

Where $\tilde{Z}(g)$ and Z (g) be the interval valued fuzzy set and fuzzy set in G.

Definition 2.4 [4] Neutrosophic set (NS)

Let U be a space of points (objects) with a generic element in U denoted by u i.e. $u \in U$. A neutrosophic set R in U is characterized by truth-membership function t_R, a indeterminacy membership function i_R and falsity-membership function f_R. Where t_R, i_R, f_R are the functions from U to $]^{-}0, 1^{+}[$ i.e. $t_R, i_R, f_R: U \to]^{-}0, 1^{+}[$ that means $t_R(u), i_R(u), f_R(u)$ are the real standard or non-standard subset of $]^{-}0, 1^{+}[$. Neutrosophic set can be expressed as $R = \{<u, (t_R(u), i_R(u), f_R(u))>: u \in U\}$. Since $t_R(u), i_R(u), f_R(u)$ are the subset of $]^{-}0, 1^{+}[$ then the sum $(t_R(u) + i_R(u) + f_R(u))$ lies between $^{-}0$ and 3^{+}, where $^{-}0 = 0 - \varepsilon$ and $3^{+} = 3 + \varepsilon$, $\varepsilon > 0$ and $\varepsilon \to 0$.

Definition 2.5 [9] Single valued neutrosophic set

Let U be a space of points (objects) with a generic element in U denoted by u. A single valued neutrosophic set H in U is expressed by $H = \{<u, (t_H(u), i_H(u), f_H(u))>, u \in U\}$, where $t_H(u), i_H(u), f_H(u): U \to [0, 1]$

Therefore for each $u \in U$, $t_H(u), i_H(u), f_H(u) \in [0, 1]$ and $0 \le t_H(u) + i_H(u) + f_H(u) \le 3$.

Definition 2.6 [4] Complement of neutrosophic set

The complement of neutrosophic set R denoted by R' and defined as $R' = \{<u, t_{R'}(u), i_{R'}(u), f_{R'}(u)>: u \in U\}$, where $t_{R'}(u) = f_R(u)$, $i_{R'}(u) = \{1^{+}\} - i_R(u)$, $f_{R'}(u) = t_R(u)$.

Definition 2.7 [8]Containment

A neutrosophic set R_1 is contained in another neutrosophic set R_2 i.e. $R_1 \subseteq R_2$ iff $t_{R_1}(u) \le t_{R_2}(u)$, $i_{R_1}(u) \le i_{R_2}(u)$ and $f_{R_1}(u) \ge f_{R_2}(u)$, $\forall u \in U$.

Definition 2.8 [4] Equality

Two single valued neutrosophic set R_1 and R_2 are equal iff $R_1 \subseteq R_2$ and $R_2 \subseteq R_1$.

Definition 2.9 [4] Union

The union of two single valued neutrosophic set R_1 and R_2 is a neutrosophic set R_3 (say) written as $R_3 = R_1 \cup R_2$.

$t_{R_3}(u) = \max\{t_{R_1}(u), t_{R_2}(u)\}$, $i_{R_3}(u) = \max\{i_{R_1}(u), i_{R_2}(u)\}$, $f_{R_1}(u) = \min\{f_{R_1}(u), f_{R_2}(u)\}$, $\forall u \in U$.

Definition 2.10 [4] Intersection

The intersection of two single valued neutrosophic set R_1 and R_2 denoted by R_4 and written as $R_4 = R_1 \cap R_2$ defined by $t_{R_4}(u) = \min\{t_{R_1}(u), t_{R_2}(u)\}$, $i_{R_4}(u) = \min\{i_{R_1}(u), i_{R_2}(u)\}$

$f_{R_4}(u) = \max\{f_{R_1}(u), f_{R_2}(u)\}$, $\forall u \in U$.

Definition 2.11 [114] Interval neutrosophic set (INS)

Let G be a non-empty set. An interval neutrosophic set $\tilde{G}$ in G is characterized by truth-membership function $t_{\tilde{G}}$, the indeterminacy function $i_{\tilde{G}}$ and falsity membership function $f_{\tilde{G}}$. For each $g \in G$, $t_{\tilde{G}}(g), i_{\tilde{G}}(g), f_{\tilde{G}}(g) \subseteq [0, 1]$ and $\tilde{G}$ defined as

$\tilde{G} = \{<g; [t^{-}_{\tilde{G}}(g), t^{+}_{\tilde{G}}(g)], [i^{-}_{\tilde{G}}(g), i^{+}_{\tilde{G}}(g)], [f^{-}_{\tilde{G}}(g), f^{+}_{\tilde{G}}(g)]: g \in G\}$.

Definition 2.12 [114] Containment

Let G_1 and G_2 be two interval neutrosophic set defined by $\tilde{G}_1 = \{<g, [t^{-}_{\tilde{G}_1}(g), t^{+}_{\tilde{G}_1}(g)], [i^{-}_{\tilde{G}_1}(g), i^{+}_{\tilde{G}_1}(g)], [f^{-}_{\tilde{G}_1}(g), f^{+}_{\tilde{G}_1}(g)]>: g \in G\}$

and $\tilde{G}_2 = \{<g, [t^{-}_{\tilde{G}_2}(g), t^{+}_{\tilde{G}_2}(g)], [i^{-}_{\tilde{G}_2}(g), i^{+}_{\tilde{G}_2}(g)], [f^{-}_{\tilde{G}_2}(g), f^{+}_{\tilde{G}_2}(g)]>: g \in G\}$

then, (i) $\tilde{G}_1 \subseteq \tilde{G}_2$ defined as

$t^{-}_{\tilde{G}_1}(g) \le t^{-}_{\tilde{G}_2}(g)$, $t^{+}_{\tilde{G}_1}(g) \le t^{+}_{\tilde{G}_2}(g)$

$i^{-}_{\tilde{G}_1}(g) \le i^{-}_{\tilde{G}_2}(g)$, $i^{+}_{\tilde{G}_1}(g) \le i^{+}_{\tilde{G}_2}(g)$

$f^{-}_{\tilde{G}_1}(g) \ge f^{-}_{\tilde{G}_2}(g)$, $f^{+}_{\tilde{G}_1}(g) \ge f^{+}_{\tilde{G}_2}(g)$ for all $g \in G$.

Definition 2.13 [114] Equality

$\tilde{G}_1 = \tilde{G}_2$ iff $\tilde{G}_1 \subseteq \tilde{G}_2$ and $\tilde{G}_2 \subseteq \tilde{G}_1$ that means $t^{-}_{\tilde{G}_1}(g) = t^{-}_{\tilde{G}_2}(g)$, $t^{+}_{\tilde{G}_1}(g) = t^{+}_{\tilde{G}_2}(g)$, $i^{-}_{\tilde{G}_1}(g) = i^{-}_{\tilde{G}_2}(g)$, $i^{+}_{\tilde{G}_1}(g) = i^{+}_{\tilde{G}_2}(g)$, $f^{-}_{\tilde{G}_1}(g) = f^{-}_{\tilde{G}_2}(g)$, $f^{+}_{\tilde{G}_1}(g) = f^{+}_{\tilde{G}_2}(g)$ for all $g \in G$.

Definition 2.14 [114] Compliment

Compliment of an interval neutrosophic set $\tilde{G}_1$ denoted by $\tilde{G}_1'$ and defined by

$\tilde{G}_1' = \{<g, [t^{-}_{\tilde{G}_1'}(g), t^{+}_{\tilde{G}_1'}(g)], [i^{-}_{\tilde{G}_1'}(g), i^{+}_{\tilde{G}_1'}(g)], [f^{-}_{\tilde{G}_1'}(g), f^{+}_{\tilde{G}_1'}(g)]>: g \in G\}$, Where, $t^{-}_{\tilde{G}_1'}(g) = f^{-}_{\tilde{G}_1}(g)$, $t^{+}_{\tilde{G}_1'}(g) = f^{+}_{\tilde{G}_1}(g)$, $i^{-}_{\tilde{G}_1'}(g) = \{1\} - i^{-}_{\tilde{G}_1}(g)$, $i^{+}_{\tilde{G}_1'}(g) = \{1\} - i^{+}_{\tilde{G}_1}(g)$,

$f^{-}_{\tilde{G}_1'}(g) = t^{-}_{\tilde{G}_1}(g)$, $f^{+}_{\tilde{G}_1'}(g) = f^{+}_{\tilde{G}_1}(g)$.

Definition 2.15 [114] Union

The union of two interval neutrosophic sets $\tilde{G}_1$, and $\tilde{G}_2$ is denoted by $\tilde{G}_3 = \tilde{G}_1 \cup \tilde{G}_2$ and defined as

$\tilde{G}_3$ = {<g, [max { $t^{-}_{\tilde{G}_1}$ (g), $t^{-}_{\tilde{G}_2}$ (g)},max { $t^{+}_{\tilde{G}_1}$ (g), $t^{+}_{\tilde{G}_2}$ (g)}], [max { $i^{-}_{\tilde{G}_1}$ (g), $i^{-}_{\tilde{G}_2}$ (g)}, max { $i^{+}_{\tilde{G}_1}$ (g), $i^{+}_{\tilde{G}_2}$ (g)}], [min { $f^{-}_{\tilde{G}_1}$ (g), $f^{-}_{\tilde{G}_2}$ (g)}, min { $f^{+}_{\tilde{G}_1}$ (g), $f^{+}_{\tilde{G}_2}$ (g)}]>: g $\in$ G}.

Definition 2.16 [114] Intersection

The intersection of two interval neutrosophic set $\tilde{G}_1$, $\tilde{G}_2$ is denoted by $\tilde{G}_4 = \tilde{G}_1 \cap \tilde{G}_2$ and defined as

$\tilde{G}_4$ = {<g, [min { $t^{-}_{\tilde{G}_1}$ (g), $t^{-}_{\tilde{G}_2}$ (g)},min { $t^{+}_{\tilde{G}_1}$ (g), $t^{+}_{\tilde{G}_2}$ (g)}], [min { $i^{-}_{\tilde{G}_1}$ (g), $i^{-}_{\tilde{G}_2}$ (g)}, min { $i^{+}_{\tilde{G}_1}$ (g), $i^{+}_{\tilde{G}_2}$ (g)}], [max { $f^{-}_{\tilde{G}_1}$ (g), $f^{-}_{\tilde{G}_2}$ (g)}, max { $f^{+}_{\tilde{G}_1}$ (g), $f^{+}_{\tilde{G}_2}$ (g)}]>: g $\in$ G}.

Definition 2.17 [95] Neutrosophic cubic set (NCS)

A neutrosophic cubic set Q (N) in a universal set G is defined as

Q (N) = {<g, $\tilde{G}$ (g), R (g)>: g $\in$ G}, where $\tilde{G}$ is an interval neutrosophic set and R is a neutrosophic set in G. In this paper,we represent neutrosophic cubic set in the following form:

Q (N) = < $\tilde{G}$, R > as order pair, set of all neutrosophic cubic sets in G, we denote it by NCS (G).

Definition 2.18 Another definition of neutrosophic cubic set

Let G be a universal set, then the neutrosophic cubic set Q (N) in G is expressed as the pair

< $\tilde{G}$, R > , where $\tilde{G}$ and R be the mappings represented by $\tilde{G}$: G $\to$ INS (G), R: $\to$ NS (G)

Combining the two mappings, NCS can be expressed as Q (N) = $\tilde{G}^R$: G $\to$ [INS (G), NS (G)] and defined as Q (N) = $\tilde{G}^R$ = {< g/ < $\tilde{G}$ (g), R (g)>>: g $\in$ G}.

Definition 2.19 [95] Containment

Let Q_1 (N) = ($\tilde{G}_1^{R_1}$) and Q_2 (N) = ($\tilde{G}_2^{R_2}$) be any two NCSs in G, then Q_1 (N) contained in Q_2 (N) i.e. Q_1 (N) $\subseteq Q_2$ (N) iff $\tilde{G}_1 \subseteq \tilde{G}_2$ and $R_1 \subseteq R_2$.

Definition 2.20 [95] Equality

Assume that Q_1 (N) = ($\tilde{G}_1^{R_1}$) and Q_2 (N) = ($\tilde{G}_2^{R_2}$) be the two NCSs in G. They are said to be equal iff Q_1 (N) $\subseteq$ Q_2 (N) and Q_2 (N) $\subseteq$ Q_1 (N) that means $\tilde{G}_1 = \tilde{G}_2$ and $R_1 = R_2$.

Definition 2.21 [95] Union

The union of two NCSs Q_1 (N) = ($\tilde{G}_1^{R_1}$) and Q_2 (N) = ($\tilde{G}_2^{R_2}$) in G is denoted by

Q_1 (N) $\cup$ Q_2 (N) = Q_3 (N) (say) and defined as

Q_3 (N) = {<g, ($\tilde{G}_1 \cup \tilde{G}_2$) (g), ($R_1 \cup R_2$) (g)>: g $\in$ G}.

Definition 2.22 [95] Intersection

The intersection of two NCS Q_1 (N) = ($\tilde{G}_1^{R_1}$) and Q_2 (N) = ($\tilde{G}_2^{R_2}$) in G is denoted by Q_1 (N) $\cup$ Q_2 (N) = Q_4 (N) (say) and defined as Q_4 (N) = {<g, ($\tilde{G}_1 \cap \tilde{G}_2$) (g), ($R_1 \cap R_2$) (g)>: g $\in$ G}.

Definition 2.23 [95]Complement

Let Q_1 (N) be a NCS. Then complement of Q_1 (N) is denoted by Q'_1 (N) = {<g, $\tilde{G}_1'$ (g), $\tilde{R}_1'$ (g)>: g $\in$ G}.

3 Similarity measure of NCS

We define similarity measure for neutrosophic cubic set.

Definition3.1

Let Q_1 and Q_2 be two NCSs in G. Similarity measure for $Q_1$and Q_2 is defined as a mapping

SM: NCS (G) × NCS (G) $\to$ [0, 1] that satisfies the following conditions:

(1) $0 \le$ SM (Q_1, Q_2) ≤ 1
(2) SM (Q_1, Q_2) = 1 iff $Q_1 = Q_2$
(3) SM (Q_1, Q_2) = SM (Q_2, Q_1)
(4) If $Q_1 \subseteq Q_2 \subseteq Q_3$ then SM (Q_1, Q_3) $\le$ SM (Q_1, Q_2) and SM (Q_1, Q_3) $\le$ SM (Q_2, Q_3) for all Q_1, Q_2, Q_3 $\in$ NCS (G).

Similarity measure for two NCSs Q_1 and Q_2 expressed as

$$SM(Q_1, Q_2) = \frac{1}{n}\sum_{i=1}^{n}\left(1-\frac{D_i}{9}\right),$$

where D_i = ($|t^{-}_{\tilde{G}_1}(g_i) - t^{-}_{\tilde{G}_2}(g_i)| + |t^{+}_{\tilde{G}_1}(g_i) - t^{+}_{\tilde{G}_2}(g_i)| + |i^{-}_{\tilde{G}_1}(g_i) - i^{-}_{\tilde{G}_2}(g_i)| + |i^{+}_{\tilde{G}_1}(g_i) - i^{+}_{\tilde{G}_2}(g_i)| + |f^{-}_{\tilde{G}_1}(g_i) - f^{-}_{\tilde{G}_2}(g_i)| + |f^{+}_{\tilde{G}_1}(g_i) - f^{+}_{\tilde{G}_2}(g_i)| + |t_{R_1}(g_i) - t_{R_2}(g_i)| + |i_{R_1}(g_i) - i_{R_2}(g_i)| + |f_{R_1}(g_i) - f_{R_2}(g_i)|$).

We now prove that the similarity measure satisfies the four stated conditions:

(1) $0 \le$ SM (Q_1, Q_2) ≤ 1

Proof: If D_i has extreme value i.e. $D_i = 0$ or 9, then SM $(Q_1, Q_2) = 1$ or 0 (1)

If D_i lies between 0 and 9 i.e$0<D_i<9$, then $0<\frac{D_i}{9}<1$

$\Rightarrow 0> -\frac{D_i}{9} > -1$

Adding 1 each part of the above inequality, we obtain

$0< 1-\frac{D_i}{9}<1$

$$\frac{1}{n}\sum_{i=1}^{n}0<\frac{1}{n}\sum_{i=1}^{n}(1-\frac{D_i}{9})<\frac{1}{n}\sum_{i=1}^{n}1=1$$

$$\Rightarrow 0<\frac{1}{n}\sum_{i=1}^{n}(1-\frac{D_i}{9})<1$$

$\Rightarrow 0<$SM $(Q_1, Q_2) <1$ (2)

Combining (1) and (2), we get $0\le$ SM $(Q_1, Q_2)\le 1$

(2) SM $(Q_1, Q_2) = 1$ iff $Q_1 = Q_2$

Proof:

If $Q_1 = Q_2$, then $D_i = 0$ by the definition of equality.

$$\text{SM}(Q_1, Q_2)=\frac{1}{n}\sum_{i=1}^{n}(1-\frac{D_i}{9})=1.$$

(3) SM (Q_1, Q_2) = SM (Q_2, Q_1)

Proof: SM $(Q_1, Q_2)=\frac{1}{n}\sum_{i=1}^{n}(1-\frac{D_i}{9})$,

where $D_i(Q_1, Q_2) = (|t^-_{\tilde{G}_1}(g_i) - t^-_{\tilde{G}_2}(g_i)| + |t^+_{\tilde{G}1}(g_i) - t^+_{\tilde{G}_2}(g_i)| + |i^-_{\tilde{G}_1}(g_i) - i^-_{\tilde{G}_2}(g_i)| + |i^+_{\tilde{G}_1}(g_i) - i^+_{\tilde{G}_2}(g_i)| + |f^-_{\tilde{G}_1}(g_i) - f^-_{\tilde{G}_2}(g_i)| + |f^+_{\tilde{G}_1}(g_i) - f^+_{\tilde{G}_2}(g_i)| + |t_{R_1}(g_i) - t_{R_2}(g_i)| + |i_{R_1}(g_i) - i_{R_2}(g_i)| + |f_{R_1}(g_i) - f_{R_2}(g_i)|)$

since, $|t^-_{\tilde{G}_1}(g_i) - t^-_{\tilde{G}_2}(g_i)| = |t^-_{\tilde{G}_2}(g_i) - t^-_{\tilde{G}_1}(g_i)|$, $|t^+_{\tilde{G}1}(g_i) - t^+_{\tilde{G}_2}(g_i)| = |t^+_{\tilde{G}2}(g_i) - t^+_{\tilde{G}_1}(g_i)|$, $|i^-_{\tilde{G}_1}(g_i) - i^-_{\tilde{G}_2}(g_i)| = |i^-_{\tilde{G}_2}(g_i) - i^-_{\tilde{G}_1}(g_i)|$, $|i^+_{\tilde{G}_1}(g_i) - i^+_{\tilde{G}_2}(g_i)| = |i^+_{\tilde{G}_2}(g_i) - i^+_{\tilde{G}_1}(g_i)|$, $|f^-_{\tilde{G}_1}(g_i) - f^-_{\tilde{G}_2}(g_i)| = |f^-_{\tilde{G}_2}(g_i) - f^-_{\tilde{G}_1}(g_i)|$, $|f^+_{\tilde{G}_1}(g_i) - f^+_{\tilde{G}_2}(g_i)| = |f^+_{\tilde{G}_2}(g_i) - f^+_{\tilde{G}_1}(g_i)|$, $|t_{R_1}(g_i) - t_{R_2}(g_i)| = |t_{R_2}(g_i) - t_{R_1}(g_i)|$, $|i_{R_1}(g_i) - i_{R_2}(g_i)| = |i_{R_2}(g_i) - i_{R_1}(g_i)|$, $|f_{R_1}(g_i) - f_{R_2}(g_i)| = |f_{R_2}(g_i) - f_{R_1}(g_i)|$.

$\Rightarrow D_i(Q_1, Q_2) = D_i(Q_2, Q_1)$

Therefore, SM (Q_1, Q_2) = SM (Q_2, Q_1).

(4) If $Q_1 \subseteq Q_2 \subseteq Q_3$, then SM $(Q_1, Q_3)\le$ SM (Q_1, Q_2) and SM $(Q_1, Q_3)\le$ SM (Q_2, Q_3) for all $Q_1, Q_2, Q_3 \in$ NCS (G).

Proof:

Let $Q_1 \subseteq Q_2 \subseteq Q_3$ then,

$t^-_{\tilde{G}_1}(g_i) \le t^-_{\tilde{G}_2}(g_i) \le t^-_{\tilde{G}_3}(g_i)$, $t^+_{\tilde{G}1}(g_i) \le t^+_{\tilde{G}_2}(g_i) \le t^+_{\tilde{G}_3}(g_i)$, $i^-_{\tilde{G}_1}(g_i) \le i^-_{\tilde{G}_2}(g_i) \le i^-_{\tilde{G}_3}(g_i)$ $i^+_{\tilde{G}_1}(g_i) \le i^+_{\tilde{G}_2}(g_i), \le i^+_{\tilde{G}_3}(g_i)$, $f^-_{\tilde{G}_1}(g_i) \ge f^-_{\tilde{G}_2}(g_i) \ge f^-_{\tilde{G}_3}(g_i)$, $f^+_{\tilde{G}_1}(g_i) \ge f^+_{\tilde{G}_2}(g_i) \ge f^+_{\tilde{G}_3}(g_i)$ $t_{R_1}(g_i) \le t_{R_2}(g_i) \le t_{R_3}(g_i)$, $i_{R_1}(g_i) \le i_{R_2}(g_i) \le i_{R_3}(g_i)$, $f_{R_1}(g_i) \ge f_{R_2}(g_i) \ge f_{R_3}(g_i)$ (3)

Now $D_i(Q_1, Q_2) = (|t^-_{\tilde{G}_1}(g_i) - t^-_{\tilde{G}_2}(g_i)| + |t^+_{\tilde{G}1}(g_i) - t^+_{\tilde{G}_2}(g_i)| + |i^-_{\tilde{G}_1}(g_i) - i^-_{\tilde{G}_2}(g_i)| + |i^+_{\tilde{G}_1}(g_i) - i^+_{\tilde{G}_2}(g_i)| + |f^-_{\tilde{G}_1}(g_i) - f^-_{\tilde{G}_2}(g_i)| + |f^+_{\tilde{G}_1}(g_i) - f^+_{\tilde{G}_2}(g_i)| + |t_{R_1}(g_i) - t_{R_2}(g_i)| + |i_{R_1}(g_i) - i_{R_2}(g_i)| + |f_{R_1}(g_i) - f_{R_2}(g_i)|)$

And $D_i(Q_1, Q_3) = (|t^-_{\tilde{G}_1}(g_i) - t^-_{\tilde{G}_3}(g_i)| + |t^+_{\tilde{G}1}(g_i) - t^+_{\tilde{G}_3}(g_i)| + |i^-_{\tilde{G}_1}(g_i) - i^-_{\tilde{G}_3}(g_i)| + |i^+_{\tilde{G}_1}(g_i) - i^+_{\tilde{G}_3}(g_i)| + |f^-_{\tilde{G}_1}(g_i) - f^-_{\tilde{G}_3}(g_i)| + |f^+_{\tilde{G}_1}(g_i) - f^+_{\tilde{G}_3}(g_i)| + |t_{R_1}(g_i) - t_{R_2}(g_i)| + |i_{R_1}(g_i) - i_{R_3}(g_i)| + |f_{R_1}(g_i) - f_{R_3}(g_i)|)$

From (3), we conclude that

$D_i(Q_1, Q_3) \ge D_i(Q_1, Q_2)$

$$\Rightarrow \frac{D_i(Q_1,Q_3)}{9} \ge \frac{D_i(Q_1,Q_2)}{9}$$

$$\Rightarrow -\frac{D_i(Q_1,Q_3)}{9} \le -\frac{D_i(Q_1,Q_2)}{9}$$

$$\Rightarrow [1-\frac{D_i(Q_1,Q_3)}{9}] \le [1-\frac{D_i(Q_1,Q_2)}{9}]$$

$$\Rightarrow \frac{1}{n}\sum_{i=1}^{n}[1-\frac{D_i(Q_1,Q_3)}{9}] \le \frac{1}{n}\sum_{i=1}^{n}[1-\frac{D_i(Q_1,Q_3)}{9}]$$

$\Rightarrow$ SM $(Q_1, Q_3)\le$ SM (Q_1, Q_2)

Similarly we can shows that SM $(Q_1, Q_3)\le$ SM (Q_2, Q_3), hence the proof.

4 MCGDM methods based on similarity measure in NCS environment

In this section we propose a new MCGDM method based on similarity measure in NCS environment. Assume that

$\alpha=\{\alpha_1,\alpha_2,\alpha_3,...,\alpha_n\}$ be a set of n alternatives with criteria $\beta=\{\beta_1,\beta_2,\beta_3,...,\beta_m\}$ and $\gamma=\{\gamma_1,\gamma_2,\gamma_3,...,\gamma_r\}$ be the r decision makers. Let $\Psi=\{\Psi_1,\Psi_2,\Psi_3,...,\Psi_r\}$ be the weight vestor of decision makers, where $\Psi_k>0$ and $\sum_{k=1}^{r}\Psi_k=1$. Proposed MCGDM method is presented using the following steps.

Step1. Formation of ideal NCS decision matrix

Ideal NCS decision matrix is an important matrix for similarity measure of MCGDM. Here we construct an ideal NCS matrix in the form

$$M=\begin{pmatrix} & \beta_1 & \beta_2 & \dots\dots & \beta_m \\ \alpha_1 & Q_{11} & Q_{12} & \dots & Q_{1m} \\ \alpha_2 & Q_{21} & Q_{22} & & Q_{2m} \\ . & . & & \dots & . \\ \alpha_n & Q_{n1} & Q_{n2} & \dots & .Q_{nm} \end{pmatrix} \quad (4)$$

Where $Q_{ij}=<G_{ij}, R_{ij}>$, i = 1, 2, 3,…, n. j = 1, 2, 3, …, m.

Step 2. Construction of NCS decision matrix

Since r decision makers are involved in the decision making process, the k-th (k = 1, 2, 3,…, r) decision maker provides the evaluation information of the alternative α_i (i= 1, 2, 3,…, n) with respect to criteria β_j (j= 1, 2, 3,…, m) in terms of the NCS. The k-th decision matrix denoted by M^k (See eq. (5)) is constreucted as follows:

$$M^k=<Q_{ij}^k>=\begin{pmatrix} & \beta_1 & \beta_2 & \dots\dots & \beta_m \\ \alpha_1 & Q_{11}^k & Q_{12}^k & \dots & Q_{1m}^k \\ \alpha_2 & Q_{21}^k & Q_{22}^k & & Q_{2m}^k \\ . & . & & \dots & . \\ \alpha_n & Q_{n1}^k & Q_{n2}^k & \dots & .Q_{nm}^k \end{pmatrix} \quad (5)$$

Where k = 1, 2, 3,…, r. i = 1, 2, 3,…, n. j = 1, 2, 3,…, m.

Step 3. Determination of attribute weight

All attribute are not equally important in decision making situation. Every decision maker provides their own opinion regarding to the attribute weight in terms of linguistic variables that can be converted into NCS. Let $w_k(\beta_j)$ be the attribute weight for the attribute β_j given by the k-th decision maker in term of NCS. We convert $w_k(\beta_j)$ into fuzzy number as follows:

$$w_k^F(\beta_j)=\begin{Bmatrix} (1-\sqrt{\frac{V_{kj}}{9}}), \text{if } \beta_j\in\beta \\ 0 \qquad \text{otherwise} \end{Bmatrix} \quad (6)$$

where V_{kj} =

$$\sqrt{\begin{Bmatrix} (1-t_k^-(\beta_j))^2+(1-t_k^+(\beta_j))^2+(i_k^-(\beta_j))^2+(i_k^+(\beta_j))^2 \\ +(f_k^-(\beta_j))^2+(f_k^+(\beta_j))^2+(1-t_k(\beta_j))^2 \\ +(i_k(\beta_j))^2+(f_k(\beta_j))^2 \end{Bmatrix}}.$$

Then aggregate weight for the criteria β_j can be determined as:

$$W_j=\frac{(1-\prod_{k=1}^{r}(1-w_k^F(\beta_j))}{\sum_{k=1}^{r}(1-\prod_{k=1}^{r}(1-w_k^F(\beta_j))} \quad (7)$$

Here $\sum_{k=1}^{r}W_j=1$.

Step 4. Calculation of weighted similarity measure

We now calculate weighted similarity measure between idel matrix M and M^k as follows:

$$S^w(M,M^k)=\langle\lambda_i^k\rangle$$

$$=\left(\lambda_1^k,\lambda_2^k,...,\lambda_n^k\right)^T=\left(\frac{1}{m}\sum_{j=1}^{m}(1-\frac{D_{ij}^k}{9})W_j\right)_{i=1}^{n} \quad (8)$$

Here, k =1, 2, 3,…, r.

Step 5. Ranking of alternatives

In order to rank alternatives, we propose the formula (see eq.9):

$$\rho_i=\sum_{k=1}^{r}\psi_k\lambda_i^k \quad (9)$$

We arrange alternatives according to the descending order values of ρ_i. The highest value of ρ_i (i= 1, 2, 3,…, n) reflects the best alternative.

5 Numerical example

We solve a MCGDM problem adapted from [108] to demonstrate the applicability and effectiveness of the proposed method. Assume that an investment company wants to invest a sum of money in the best option. The investment company forms a decision making committee comprising of three members (k_1, k_2, k_3) to make a panel of four alternatives to invest money. The alternatives are Car company (α_1), Food company (α_2), Computer company

(α_3) and Arm company (α_4). Decision makers take decision based on the criteria namely, risk analysis (β_1), growth analysis (β_2), environment impact (β_3) and criterion weights are provided by the decision makers in terms of linguistic variables that can be converted into NCS.(See Table 1).

Table 1: Linguistic term for rating of attribute/ criterion

Linguistic terms	NCS
Very important (VI)	<[.7, .9], [.1, .2], [.1, .2], (.9, .2,.2)>
Important (I)	<[.6, .8], [.2, .3], [.2, .4], (.8, .3, .4)>
Medium (M)	<[.4, .5], [.4, .5], [.4, .5], (.5, .5, .5)>
Unimportant (UI)	<[.3, .4], [.5, .6], [.5, .7], (.4, .6, .7)>
Very unimportant (VUI)	<[.1, .2], [.6, .8], [.7, .9], (.2, .8, .9)>

Step1. Formation of ideal NCS decision matrix

We construct ideal NCS decision matrix (see eq.(10).

$$M=\begin{pmatrix} & \beta_1 & \beta_2 & \beta_3 \\ \alpha_1 & <[1,1],[0,0],[0,0],(1,0,0)> & <[1,1],[0,0],[0,0],(1,0,0)> & <[1,1],[0,0],[0,0],(1,0,0)> \\ \alpha_2 & <[1,1],[0,0],[0,0],(1,0,0)> & <[1,1],[0,0],[0,0],(1,0,0)> & <[1,1],[0,0],[0,0],(1,0,0)> \\ \alpha_3 & <[1,1],[0,0],[0,0],(1,0,0)> & <[1,1],[0,0],[0,0],(1,0,0)> & <[1,1],[0,0],[0,0],(1,0,0)> \\ \alpha_4 & <[1,1],[0,0],[0,0],(1,0,0)> & <[1,1],[0,0],[0,0],(1,0,0)> & <[1,1],[0,0],[0,0],(1,0,0)> \end{pmatrix} \quad (10)$$

Step 2. Construction of NCS decision matrix

The NCS decision matrices are constructed for four alternatives with respect to the three criteria.

Decision matrix for k_1 in NCS form

M^1 =

$$M^1=\begin{pmatrix} & \beta_1 & \beta_2 & \beta_3 \\ \alpha_1 & <[.7,.9],[.1,.2],[.1,.2],(.9,.2,.2)> & <[.7,.9],[.1,.2],[.1,.2],(.9,.2,.2)> & <[.4,.5],[.4,.5],[.4,.5],(.5,.5,.5)> \\ \alpha_2 & <[.6,.8],[.2,.3],[.2,.4],(.8,.3,.4)> & <[.4,.5],[.4,.5],[.4,.5],(.5,.5,.5)> & <[.7,.9],[.1,.2],[.1,.2],(.9,.2,.2)> \\ \alpha_3 & <[.4,.5],[.4,.5],[.4,.5],(.5,.5,.5)> & <[.6,.8],[.2,.3],[.2,.4],(.8,.3,.4)> & <[.4,.5],[.4,.5],[.4,.5],(.5,.5,.5)> \\ \alpha_4 & <[.3,.4],[.5,.6],[.5,.7],(.4,.6,.7)> & <[.4,.5],[.4,.5],[.4,.5],(.5,.5,.5)> & <[.7,.9],[.1,.2],[.1,.2],(.9,.2,.2)> \end{pmatrix}$$

Decision matrix for k_2 in NCS form

M^2 =

$$\begin{pmatrix} & \beta_1 & \beta_2 & \beta_3 \\ \alpha_1 & <[.3,.4],[.5,.6],[.5,.7],(.4,.6,.7)> & <[.4,.5],[.4,.5],[.4,.5],(.5,.5,.5)> & <[.7,.9],[.1,.2],[.1,.2],(.9,.2,.2)> \\ \alpha_2 & <[.4,.5],[.4,.5],[.4,.5],(.5,.5,.5)> & <[.4,.5],[.4,.5],[.4,.5],(.5,.5,.5)> & <[.7,.9],[.1,.2],[.1,.2],(.9,.2,.2)> \\ \alpha_3 & <[.7,.9],[.1,.2],[.1,.2],(.9,.2,.2)> & <[.7,.9],[.1,.2],[.1,.2],(.9,.2,.2)> & <[.4,.5],[.4,.5],[.4,.5],(.5,.5,.5)> \\ \alpha_4 & <[.6,.8],[.2,.3],[.2,.4],(.8,.3,.4)> & <[.4,.5],[.4,.5],[.4,.5],(.5,.5,.5)> & <[.7,.9],[.1,.2],[.1,.2],(.9,.2,.2)> \end{pmatrix}$$

Decision matrix for k_3 in NCS form

M^3 =

$$\begin{pmatrix} & \beta_1 & \beta_2 & \beta_3 \\ \alpha_1 & <[.4,.5],[.4,.5],[.4,.5],(.5,.5,.5)> & <[.4,.5],[.4,.5],[.4,.5],(.5,.5,.5)> & <[.7,.9],[.1,.2],[.1,.2],(.9,.2,.2)> \\ \alpha_2 & <[.4,.5],[.4,.5],[.4,.5],(.5,.5,.5)> & <[.7,.9],[.1,.2],[.1,.2],(.9,.2,.2)> & <[.4,.5],[.4,.5],[.4,.5],(.5,.5,.5)> \\ \alpha_3 & <[.7,.9],[.1,.2],[.1,.2],(.9,.2,.2)> & <[.6,.8],[.2,.3],[.2,.4],(.8,.3,.4)> & <[.6,.8],[.2,.3],[.2,.4],(.8,.3,.4)> \\ \alpha_4 & <[.7,.9],[.1,.2],[.1,.2],(.9,.2,.2)> & <[.4,.5],[.4,.5],[.4,.5],(.5,.5,.5)> & <[.3,.4],[.5,.6],[.5,.7],(.4,.6,.7)> \end{pmatrix}$$

Step 3. Determination of attribute weight

The linguistic terms shown in Table 1 are used to evaluate each attribute. The importance of each attribute for every decision maker is rated with linguistic terms shown in Table 2. Linguistic terms are converted into NCS (See Table 3.) .

Table 2. Attribute rating in linguistic variables

	β_1	β_2	β_3
K_1	VI	M	I
K_2	VI	VI	M
K_3	M	VI	M

Table 3. Attribute rating in NCS

	β_1	β_2	β_3
K_1	<[.7, .9], [.1, .2], [.1, .2], (.9, .2,.2)>	<[.4, .5], [.4, .5], [.4, .5], (.5, .5, .5)>	<[.6, .8], [.2, .3], [.2, .4], (.8, .3, .4)>
K_2	<[.7, .9], [.1, .2], [.1, .2], (.9, .2,.2)>	<[.7, .9], [.1, .2], [.1, .2], (.9, .2,.2)>	<[.4, .5], [.4, .5], [.4, .5], (.5, .5, .5)>
K_3	<[.4, .5], [.4, .5], [.4, .5], (.5, .5, .5)>	<[.7, .9], [.1, .2], [.1, .2], (.9, .2,.2)>	<[.4, .5], [.4, .5], [.4, .5], (.5, .5, .5)>

Using eq. (6) and eq. (7), we obtain the attribute weights as follows: $w_1 = .36, w_2 = .37, w_3 = .27$. (11)

Step 4. Calculation of weighted similarity measures

We now calculate weighted similarity measures using the formula (8).

$$S^w(M, M^1) = \begin{pmatrix} .25 \\ .22 \\ .19 \\ .24 \end{pmatrix}, S^w(M, M^2) = \begin{pmatrix} .18 \\ .20 \\ .25 \\ .22 \end{pmatrix}, S^w(M, M^3) = \begin{pmatrix} .20 \\ .21 \\ .25 \\ .20 \end{pmatrix} \quad (12)$$

Step 5. Ranking of alternatives

We rank the alternatives according to the descending value of ρ_i (i = 1, 2, 3, 4) using eq.(10), eq.(11), and eq. (12).

We obtain $\rho_1 = .202, \rho_2 = .206, \rho_3 = .232, \rho_4 = .216$, Therefore the ranking order is

$\rho_3 > \rho_4 > \rho_2 > \rho_1 \Rightarrow \alpha_3 > \alpha_4 > \alpha_2 > \alpha_1$.

Hence Computer company (α_3) is the best alternative for money investment.

6 Conclusion

In this paper we have defined similarity measure between neutrosophic cubic sets and proved its basic properties . We have developed a new multi criteria group decision making method basd on the proposed similarity measure. We also provide an illustrative example for multi criteria group decision making method to show its applicability and effectiveness. We have employed linguistic variables to present criteria weights and presented conversion of linguistic variables into neutrosophic cubic numbers. We have also proposed a conversion formula for neutrosophic cubic number into fuzzy number. The poposed method can be applied to other MCGDM making problems in neutrosophic cubic set environment such as banking system, engineering problems, school choice problems, teacher selection problem, etc. We also hope that the proposed method will open up a new direction of research work in neutrosophic cubic set environment.

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